

Numerical Optimization of the "Tornado" Combustor Aerodynamic Parameters

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Theoretical investigations of the reverse vortex combustor (RVC) with plasma stabilization have been conducted. Different effects of spatial arc on the burnout processes inside the combustor and ecological characteristics have been considered. The results of this work and recommendations can be used for the reverse vortex combustion chamber operation mode and geometry optimization, perspective burners design for propulsion and power generation.

I. Nomenclature

A	=	empirical constant
A_k	=	pre-exponential factor
C_j	=	molar concentration of each species j
$C_{\varepsilon_1}, C_{\varepsilon_2}, C_{\varepsilon_3}$	=	constants
D	=	diffusion coefficient
E_k	=	activation energy
F^s	=	rate of rise in momentum
g	=	specific body force
I	=	specific internal energy
i, j, k	=	unit vectors in the x, y, z directions
J	=	heat flux vector;
k	=	turbulent kinetic energy
L	=	length
M_m	=	molecular weight of species m
P	=	pressure
Pr_k, Pr_ε	=	Prandtl number
\dot{Q}^c, \dot{Q}^s	=	source terms due to chemical heat release and spray interactions
R	=	gas constant
R_m	=	source term due to chemistry
R_{mk}	=	rate of creation (destruction) of species m during reaction k
S_m	=	source terms due to dispersed phase
t	=	time
W^s	=	source term due to interaction with the spray
X_m	=	the mass fraction of chemical species m

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X_m^*	=	fine-scale species mass fraction after reacting over the time τ^*
β_k	=	temperature exponent
μ	=	coefficient of viscosity
v_{jk}^l	=	concentration exponent
v_{mk}	=	stoichiometric coefficient
ρ	=	mass density
ρ_m	=	mass density of species m
ρ^s	=	source term due to the spray of liquid
σ	=	viscous stress tensor
τ^*	=	time scale
ξ^*	=	fine scale

II. Introduction

CHANGES in an energy consumption structure require modification of the combustion process, and employment of low-emission, more effective flame stabilization devices. The flame stabilization method invented by Applied Plasma Technologies (APT)¹ provides the basis to build a prototype of the “ideal” combustion chamber named the “Tornado” combustor. A plasma generator, recently developed by the APT, has demonstrated dramatically extended lifetime and would be utilized as the second basic element of the Plasma Assisted Tornado Combustion system (PATC).

There are several flame stabilization methods: addition of another more reactive fuel, use of catalysts, oxygen injection, introduction of a highly heated body, or by applying other energy sources (e.g. electric, acoustic, microwave, plasma², or pre-combustion chambers). The main idea of the PATC stabilization is to establish a flame (or plasma jet) along the axis of a cylindrical combustion chamber in the opposite direction to incoming air which is strongly swirling and flowing along the chamber walls. In this case, cold gas cannot move to the inner, reverse flow zone before it loses the main part of its rotational speed. Hence, initially cold gas flows along the wall to the closed end of the cylindrical vessel, and turbulent micro-volumes of this cold gas, which lost their kinetic energy near the wall, migrate radially towards the centre. As a result, cold gas comes into the hot zone from all sides, except the outlet side, and no significant recirculation zone is formed. Working process visualizations in the PATC without fuel injection (main air flow rate 2.5 g/s, arc voltage 980 V, arc current 0.2 A) is shown in Fig. 1.

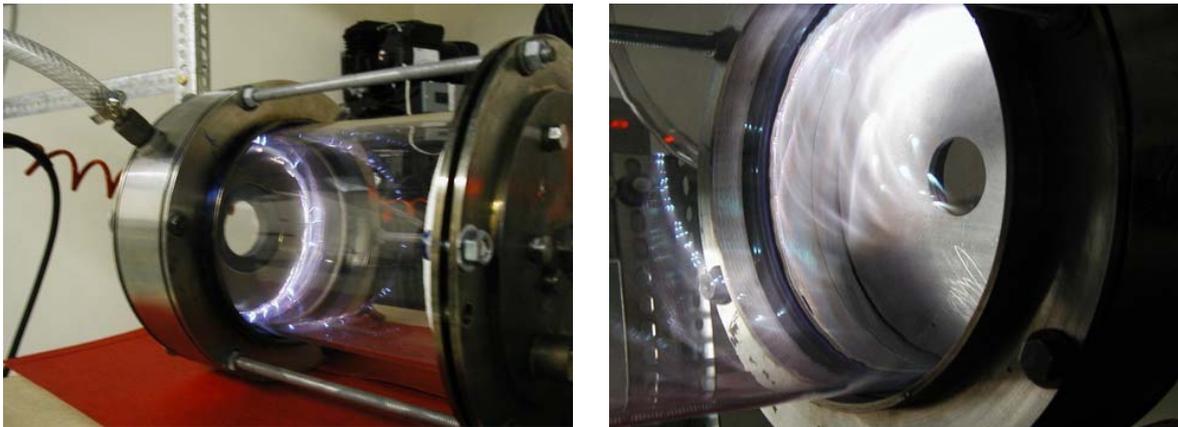


Figure 1. Working process visualizations in the “Tornado” combustor with spatial arc excited in the nozzle part.

A full-scale atmospheric pressure PATC tests proved the concept’s advantages as follows³: high efficient internal mixing of fuel and oxidizer, stable combustion with dramatically extended flammability limits, simple air swirler and fuel injectors, no cooling of the combustor walls, simple combustor design, cheaper materials for combustor fabrication, and simple conversion into the multi-fuel and multi-zone combustor. A laser Doppler

velocimetry system was used to measure the mean axial and swirl velocity components and their respective fluctuations in the "Tornado" combustor under cold, non-reacting, isothermal conditions⁴. For modeling of aerodynamic processes inside the combustor a generalized method was used based on numerical solution of the combined conservation and transport equations for turbulent system. Comparison between experimental data and computed predictions using different turbulence models has been completed.

Objectives of current research are numerical optimization of the PATC aerodynamic parameters and minimization of the chamber pressure losses. For comparison with calculated data a complete atmospheric pressure combustion system with ID = 145 mm and internal volume of 4 liters has been designed, manufactured and tested on natural gas with air flow up to 20 grams per second with different methods of fuel injection.

Fig. 2 presents the process visualization and contours of static temperature in the axial "Tornado" combustor cross-sections with central fuel injection for the main air flow rate of 17.56 g/s and total fuel equivalence ratios of 0.172, 0.35, 0.7 (at fuel flow rate 0.18, 0.366, 0.7 g/s accordingly).

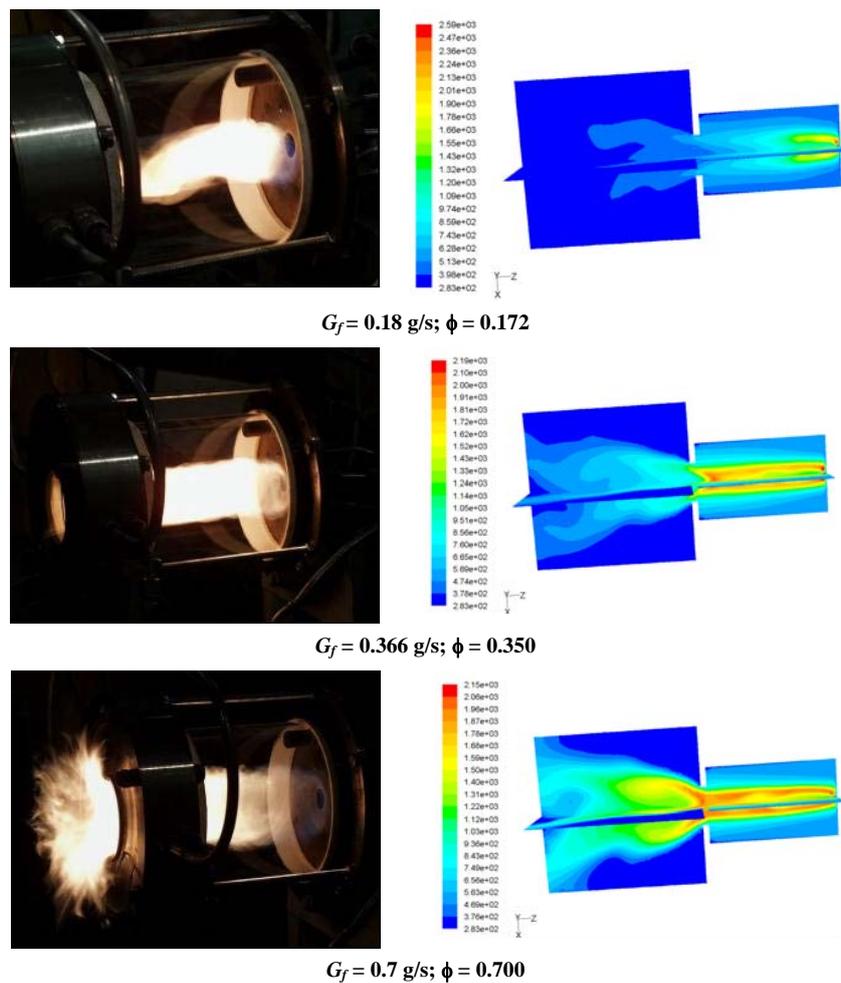


Figure 2. Process visualization and contours of static temperature in the axial "Tornado" cross-sections with central fuel injection.

Fig. 3 presents the process visualizations and contours of static temperature in the axial "Tornado" combustor cross-sections with fuel feeding through two symmetric holes on the ID = 92 mm for the air flow rate of 17.56 g/s and total fuel equivalence ratios of 0.144, 0.191, 0.287 (at fuel flow rate 0.15, 0.2, 0.3 g/s accordingly).

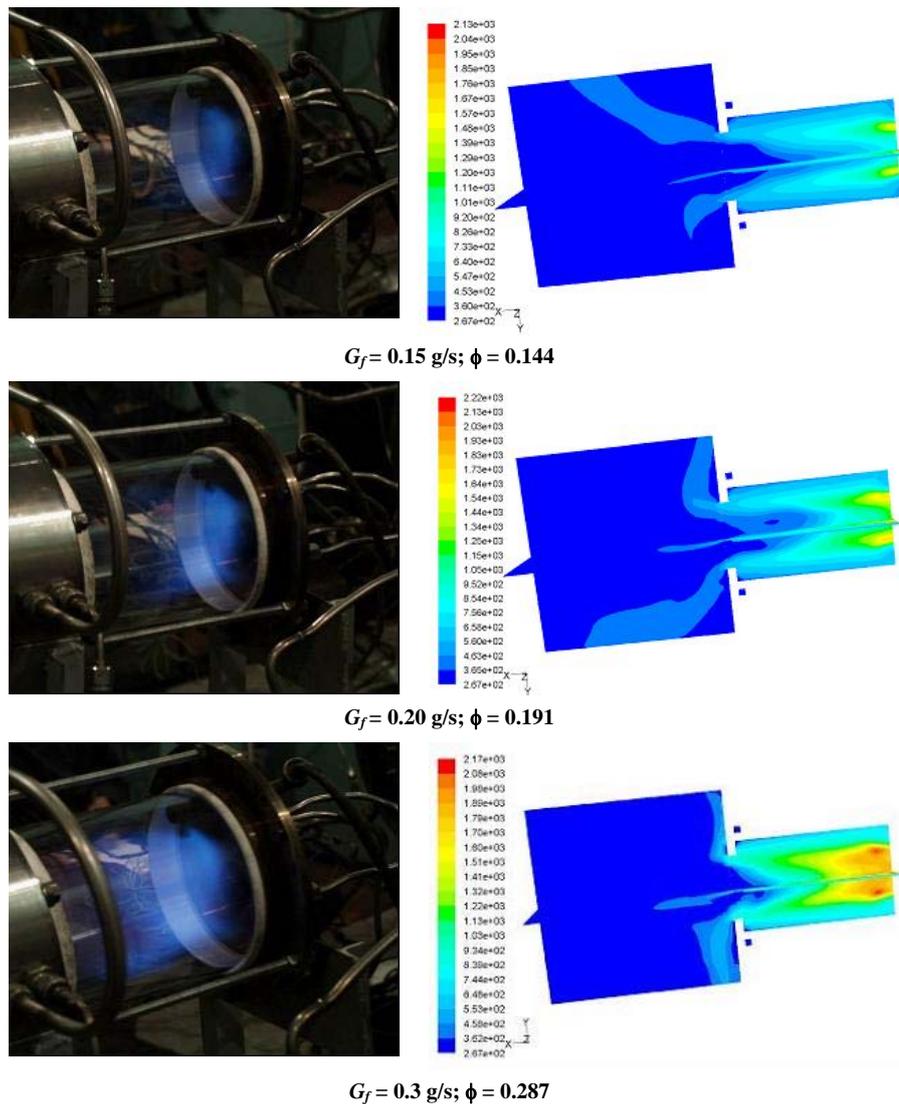


Figure 3. Process visualization and contours of static temperature in the axial “Tornado” cross-sections with 92 mm fuel injection.

Calculated data testify about complex aerodynamic flow structure in the combustor and influence of fuel injection methods on mixing and burning efficiency. Availability of recirculation flows in the exit nozzle region considerably complicates calculation and measurements of the main combustor parameters.

Offered solution combines and develops known reverse vortex flow application for the combustion process improvement and high voltage rotating spatial arc, and will result in development of the reverse vortex combustor with spatial arc. It demonstrated extremely wide range of operation parameters with lean flame outs by 0.03, maximum wall temperature by 150 °C at the exhaust gases temperature point about 900 °C, proved the concept of spatial arc application, which lighted air/fuel mixture at a power level of 40 W (lowest level for the power supply regulation at the tests time), and provided significant flame stabilization. Spatial arc was rotated by the swirling air flow starting from 50 mm of water column pressure differential and its length in some cases reached 120-145 mm. However in first prototype design spatial arc does not contact immediately with a zone of fuel preparation that slightly diminish plasma influence on burning processes.

For conducting more detailed investigations of the plasma-chemical intensification mechanisms especially in high-pressure conditions, using of more compact design is more acceptable. For this purpose new “Tornado”- pilot

has been calculated by CFD method and PATC-2 with spatial arc has been designed. It has ID = 73 mm, length 150 mm, exit nozzle diameter 50 mm, nozzle length 10 mm. Main and plasma air could be injected independently through tangential channels in the exit nozzle and bottom areas accordingly. In this case burning of non-equilibrium rotating spatial arc near the fuel injection zone will provide favorable conditions for strengthening of thermal, kinetic, and turbulent plasma impacts on processes of fuel-oxidizer mixture preparation, ignition and burning-out. Several fuel injection approaches, as (1) through symmetrically located holes at the bottom plate, (2) through central nozzle, and (3) combined distributed + central inputs have been stipulated. The bottom part of PATC-2 with spatial arc is shown in Fig. 4.

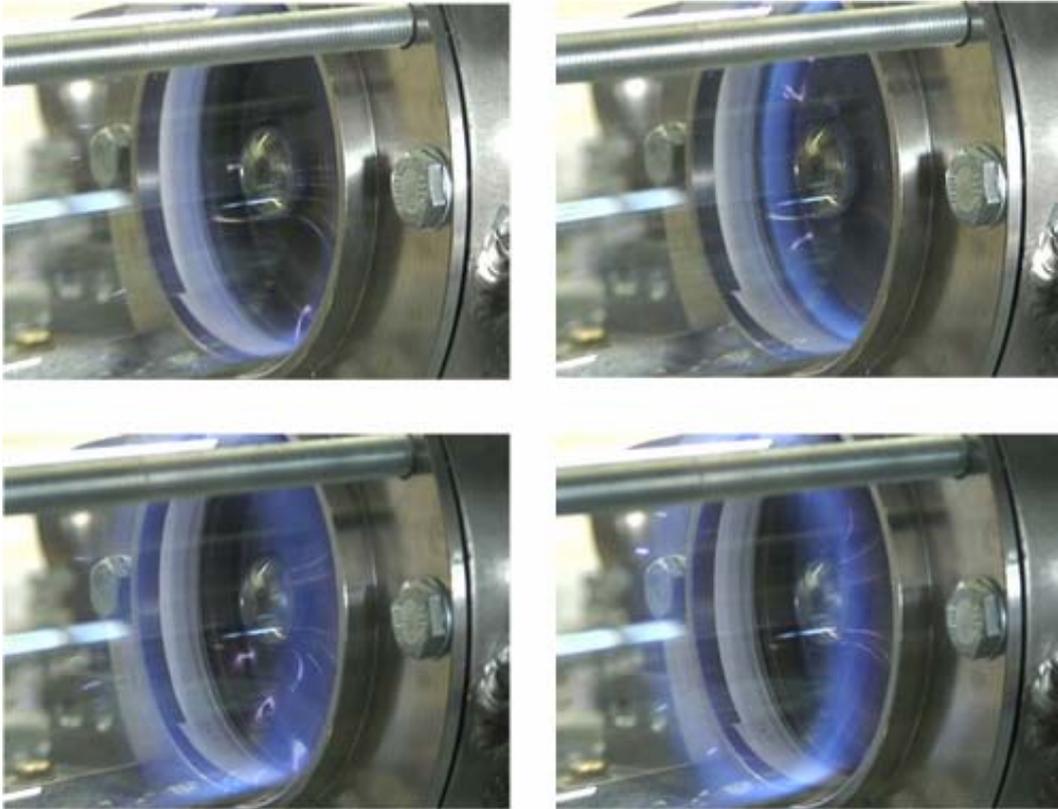


Figure 4. Visualization of PATC-2 with spatial arc in the bottom part.

A number of numerical simulations for PATC-2 were performed to investigate influence of spatial arc on chemical and physical processes inside the device.

III. Mathematical modeling

The development of a new generation of combustion chambers with spatial arc should be based on better understanding of the physical and chemical processes of turbulent combustion in highly swirled flows and ability of such combustors modeling taking in account complicity of their 3D geometry and variety of operation modes. For modeling of physical and chemical processes inside the RVC with spatial arc a generalized method based on numerical solution of the combined conservation and transport equations for multi-component chemically reactive turbulent system was employed. This method provides a procedure of the sequential numerical integration of the differential equations, which describe reacting viscous gas flows. A 3D model of stationary and non-stationary reacting flows has been utilized which allows to predict plasma-chemical influence and optimize parameters of the combustors taking into consideration mixing, turbulence, radiation and combustion features.

Modeling of physical and chemical processes in the reverse vortex combustor is based on solution of the well-known system of the differential equations of mass, impulse and energy conservation for the multi-component, turbulent, chemically reacting system⁵⁻⁸ in the following way:

- the mass conservation equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \dot{\rho}^s,$$

- the momentum conservation equation

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p - \nabla \left(\frac{2}{3} \rho k \right) + \nabla \cdot \boldsymbol{\sigma} + F^s + \rho \mathbf{g},$$

- the continuity equation for species m

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \mathbf{u}) = \nabla \cdot \left[\rho D \nabla \left(\frac{\rho_m}{\rho} \right) \right] + R_m + S_m,$$

- the internal energy equation

$$\frac{\partial (\rho I)}{\partial t} + \nabla \cdot (\rho \mathbf{u} I) = -\rho \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{J} + \rho \varepsilon + \dot{Q}^c + \dot{Q}^s,$$

- the turbulent kinetic energy transport equation

$$\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{u} k) = -\frac{2}{3} \rho k \nabla \cdot \mathbf{u} + \frac{\boldsymbol{\sigma}}{\nabla \mathbf{u}} + \nabla \cdot \left[\left(\frac{\mu}{Pr_k} \right) \nabla k \right] - \rho \varepsilon + \dot{W}^s,$$

- the dissipation rate of the turbulent kinetic energy transport equation

$$\frac{\partial \rho \varepsilon}{\partial t} + \nabla \cdot (\rho \mathbf{u} \varepsilon) = -\left(\frac{2}{3} C_{\varepsilon_1} - C_{\varepsilon_3} \right) \rho \varepsilon \nabla \cdot \mathbf{u} + \nabla \cdot \left[\left(\frac{\mu}{Pr_\varepsilon} \right) \nabla \varepsilon \right] + \frac{\varepsilon}{k} \left[C_{\varepsilon_1} \frac{\boldsymbol{\sigma}}{\nabla \mathbf{u}} - C_{\varepsilon_2} \rho \varepsilon + C_s \dot{W}^s \right].$$

The above mentioned equations are written down in the vectorial form for which operator

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z},$$

$$\mathbf{u} = u(x, y, z, t) \mathbf{i} + v(x, y, z, t) \mathbf{j} + w(x, y, z, t) \mathbf{k},$$

where i, j, k are the unit vectors in the x, y, z directions.

In these equations t is the time; ρ , the mass density of the mixture; ρ^s , the source term due to the spray of liquid; p , the fluid pressure; k , the turbulent kinetic energy; $\boldsymbol{\sigma}$, the viscous stress tensor; F^s , the rate of rise in momentum; \mathbf{g} , the specific body force; ρ_m , the mass density of species m ; D , the diffusion coefficient; R_m, S_m , the source terms due to chemistry and dispersed phase; I , the specific internal energy; \mathbf{J} , the heat flux vector; \dot{Q}^c, \dot{Q}^s , the source terms due to chemical heat release and spray interactions; μ , the coefficient of viscosity; $C_{\varepsilon_1}, C_{\varepsilon_2}, C_{\varepsilon_3}, Pr_k, Pr_\varepsilon$, constants; \dot{W}^s , the source term due to interaction with the spray.

We have to note, that in circulating flows turbulence viscosity factor is anisotropic. That's why application of a standard k - ε - turbulence model could be not efficient. For those cases the RNG k - ε - model could be applied⁹. The

RNG-based k - ε - turbulence model is derived from the instantaneous Navier-Stokes equations, using a mathematical technique called "renormalization group" (RNG) methods.

Turbulent flows in "Tornado" combustor are also characterized by eddies with a wide range of length and time scales. The largest eddies are typically comparable in size to the characteristic length of the mean flow. The smallest scales are responsible for the dissipation of turbulence kinetic energy. Therefore in some cases for definition of instantaneous velocities inside "Tornado" combustor the large eddy simulation (LES) model¹⁰⁻¹³ (Smagorinsky-Lilly dynamic model, discretization density - second order upwind, momentum - bounded central differencing, energy - second order upwind, and pressure-velocity coupling - SIMPLEC) has been used. It is assumed that momentum, mass, energy, and other passive scalars are transported mostly by large eddies. Small eddies are less dependent on the geometry, and tend to be more isotropic. It is possible, in theory, to directly resolve the whole spectrum of turbulent scales using an approach known as direct numerical simulation (DNS). In LES large eddies are resolved directly, while small eddies are modeled. The governing equations employed for LES are obtained by filtering the time-dependent Navier-Stokes equations in either Fourier (wave-number) space or configuration (physical) space. The filtering process effectively filters out eddies whose scales are smaller than the filter width or grid spacing used in the computations. The resulting equations thus govern the dynamics of large eddies.

LES model has been used in unsteady cases to check the steady solutions. Transient solution helps to understand the physical process in the central chamber vortex, where the velocity magnitudes are about zero and the flow is highly unstable.

The source of chemical species m due to reaction R_m is computed as the sum of the reaction sources over the k reactions

$$R_m = \sum_k R_{mk},$$

where R_{mk} is the rate of creation (destruction) of species m during reaction k .

Reaction rate is calculated considering both the Arrhenius, and Magnussen and Hjertager models:

$$R_{mk} = \nu_{mk} M_m T^{\beta_k} A_k \prod_j [C_j]^{\nu_{jk}^|} \exp(-E_k / RT), \quad R_{mk} = A \rho \frac{\varepsilon}{k} \frac{X_m}{\nu_{mk}},$$

where ν_{mk} is the stoichiometric coefficient; M_m , the molecular weight of species m ; β_k , the temperature exponent; A_k , the pre-exponential factor; C_j , the molar concentration of each species j ; $\nu_{jk}^|$, the concentration exponent; E_k , the activation energy; R , the gas constant; A , empirical constant; X_m , the mass fraction of chemical species m .

The reaction rate is taken to be the smaller of two these expressions. This chemistry- turbulence approach is used because there are regions within the reverse vortex combustor where the turbulent mixing rate is faster than the chemical kinetics (low temperature regions in the vicinity of the fire tube walls, axial and radial inlets).

The EDC model is an extension of the eddy-dissipation model to include detailed chemical mechanisms in turbulent flows. It assumes that reaction occurs in small turbulent structures, called the fine scales ξ^* :

$$\xi^* = C_\xi \left(\frac{\nu \varepsilon}{k} \right)^{3/4}, \quad C_\xi = 2,1377.$$

Combustion at the fine scales is assumed to occur as a constant pressure reactor, with initial conditions taken as the current species and temperature in the cell. Reactions proceed over the time scale τ^* :

$$\tau^* = C_\tau \left(\frac{\nu}{\varepsilon} \right)^{1/2}, \quad C_\tau = 0,4082,$$

governed by the Arrhenius rates, and are integrated numerically using the ISAT algorithm.

Reaction rate for this model is calculated as:

$$R_{mk} = \frac{\rho(\xi^*)^2}{\tau^* [1 - (\xi^*)^3]} (X_m^* - X_m),$$

where X_m^* is fine-scale species mass fraction after reacting over the time τ^* .

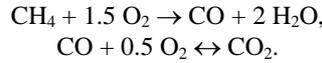
The boundary conditions in the axial and radial inlets, symmetry axes, walls and outlet from a RVC were set in accordance with the conditions for carrying out physical experiments and recommendations for modeling the turbulent burning processes. The method for the system solution, the finite difference scheme and the solution stability analysis are explained in detail in ^{2-5, 12, 13}.

Because of electric arc (discharge) simulation on the basis of complete system of the magnetic fluid dynamics equations calculation has many obstacles the present work objective is numerical definition of only kinetic and thermal spatial arc influence on gas fuel burning-out inside the combustor. For this purpose three-dimensional CFD calculations of the PATC-2 with different flow rates of main and plasma-forming air have been conducted with below accepted approach concerning spatial arc.

Spatial arc is represented by a number of some small volumes, which rotate around the chamber central axis. In this case spatial arc has been modeled using rotating reference frame approach in steady cases. This approach means that the equation of momentum conservation has the relative speed in its left part for moving volume. Relative speed depends on the angular velocity of the volume. The moving mesh was used in unsteady case with LES-turbulence model.

The spatial arc was assumed as the thermal source of approximately 100 W. That means that the source term in equation for conservation of energy $\dot{Q}^c = 7.42e + 07 \text{ W} / m^3$.

The reaction mechanism in spatial arc differs from the whole combustion volume. As a base the two-stage global mechanism of methane combustion has been chosen:



The Arrhenius rates for this mechanism are presented in Table 1. It has been modified for the volume of spatial arc for kinetic plasma influence modeling. The activation energy in the second reaction of carbon monoxide oxidation has been decreased up to 70 % comparing with the base mechanism in accordance with experimental data obtained at APT and NUS.

Table 1. Arrhenius rates for the two-step global mechanism of methane oxidation.

Reactions	A_k	E_k	β_k	v'_{jk}		
$\text{CH}_4 + 1,5\text{O}_2 \rightarrow \text{CO} + 2\text{H}_2\text{O}$	5.012e+11	2e+08	0	CH_4	0.7	O_2 0.8
$\text{CO} + 0,5\text{O}_2 \rightarrow \text{CO}_2$	2.239e+12	1.7e+08	0	CO	1	O_2 0.25

IV. Theoretical results

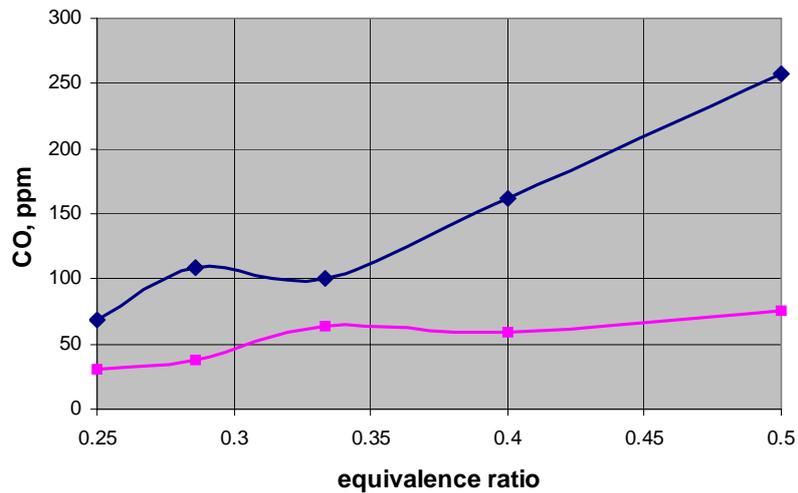
In numerical experiments three main factors of spatial arc influence on the process inside the PATC-2 have been taken into consideration: thermal, chemical and mixing.

The calculations have been carried out for five values of fuel equivalence ratios – 0.25; 0.285; 0.33; 0.4; 0.5. The overall air mass-flow rate was 4.6 grams per second. The main air quantity of 3 g/s was injected through the swirler in the region of exit nozzle and the rest of air through the bottom swirler. 90 % of the gaseous fuel (methane) was injected through symmetrically located holes in the bottom and 10 % through the central fuel nozzle.

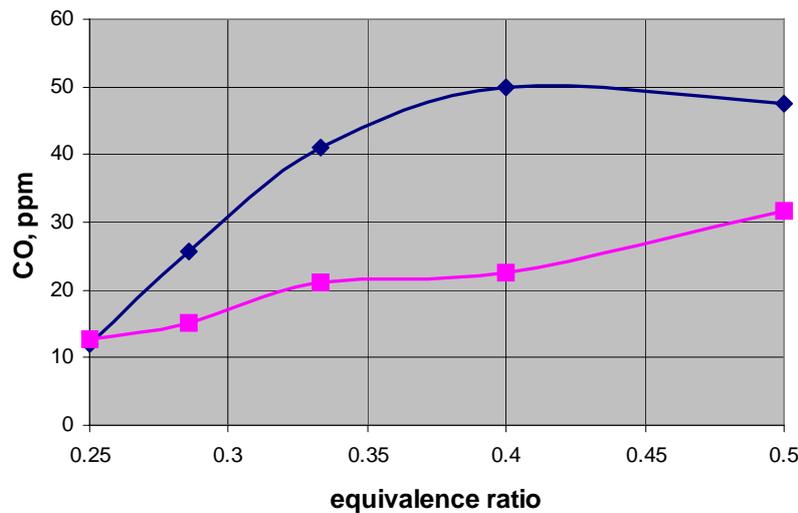
The combustion efficiency can be quite soundly determined by the emission of carbon monoxide. Dependencies between CO emission and equivalence ratio in the outlet cross-section 1 (a) and outlet cross-section 2 (b) are shown

in Fig. 5. Cross-section 1 is the plane located right at the outlet nozzle, while cross-section 2 is the plane 150 mm far from the outlet nozzle.

The dark blue curve shows the basic case without any spatial arc influence. The red curve shows impact of different plasma effects. Presence of spatial arc improves the burnout process inside combustor, especially for rich mixtures. It is necessary to pay attention that mixing caused by arc movement acts as an additional positive factor for increasing combustion efficiency. Spatial arc application allowed CO emission decrease on 55 % in cross-section 1 and on 35 % in cross-section 2 for all fuel equivalence ratios.



(a)



(b)

—◆— without spatial arc; —■— thermal and chemical effect of the spatial arc

Figure 5. Spatial arc influence on burnout process in “Tornado” combustor.

Figure 6 shows the carbon monoxide distribution along the central axis of the PATC-2 with and without spatial arc. For all cases growth of equivalence ratio leads to decreasing of the carbon monoxide concentration along the chamber. Absolute CO values decrease when plasma stabilization is used for all equivalence ratios. Besides, the length of the active CO formation zone is reduced from 60 % for rich mixtures to 35 % for lean mixtures.

Spatial arc does not significantly change temperature distribution inside the combustor because the plasma, which is used for the burning intensification, is not thermal. The arc with power of 100 W increases temperature inside the chamber only by 20-30 degrees. The main reason for temperature field variation is the equivalence ratio changing.

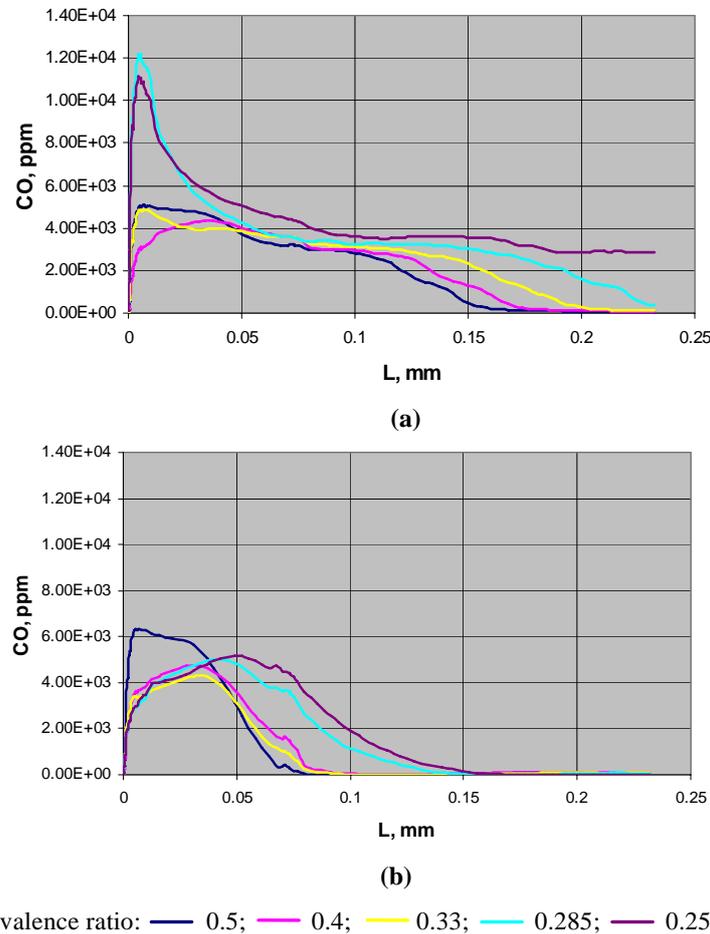


Figure 6. Spatial arc influence on burnout process along the central axis of the combustor without spatial arc (a) and with spatial arc (b).

It is necessary to investigate what action has plasma stabilization upon NO emission, because it is one of the most important ecological combustor characteristics. Three main mechanisms of NO formation: thermal, prompt and N_2O mechanism have been used.

The diagrams in Fig. 7 determine relations between equivalence ratio and NO emission in the outlet cross-sections. It is evident from the graphs that the plasma stabilization may increase NO emission on 15-25 % due to thermal influence. The main NO formation mechanism for diffusion flames (similar in PATC-2) is thermal, and the role of the prompt and other NO mechanisms are negligible. The other consequence of that fact is that NO emission rises with the growth of equivalence ratio.

Unsteady combustor aerodynamic structure prediction with LES turbulence model is the next step of the numerical experiments. These calculations allow to find values of parameter fluctuations in the combustor that is necessary to insure burning process stability in the PATC-2. The calculation time (0.042 s) has been chosen to obtain a complete revolution of spatial arc. As LES-modeling requires additional computational resources and because of lean mixtures are more unstable only case with equivalence ratio 0.25 has been investigated. In Fig. 8 time fluctuations of exhaust CO emission (a) and temperature (b) at the outlet cross-section 1 are shown. Large initial parameter fluctuations are induced by discrepancies of initial conditions, but the numerical solution becomes

more stable starting from 0.015 s, and fluctuations of all parameters decrease. The average values of exhaust temperature and CO emission are 905 K and 2.2 ppm accordingly.

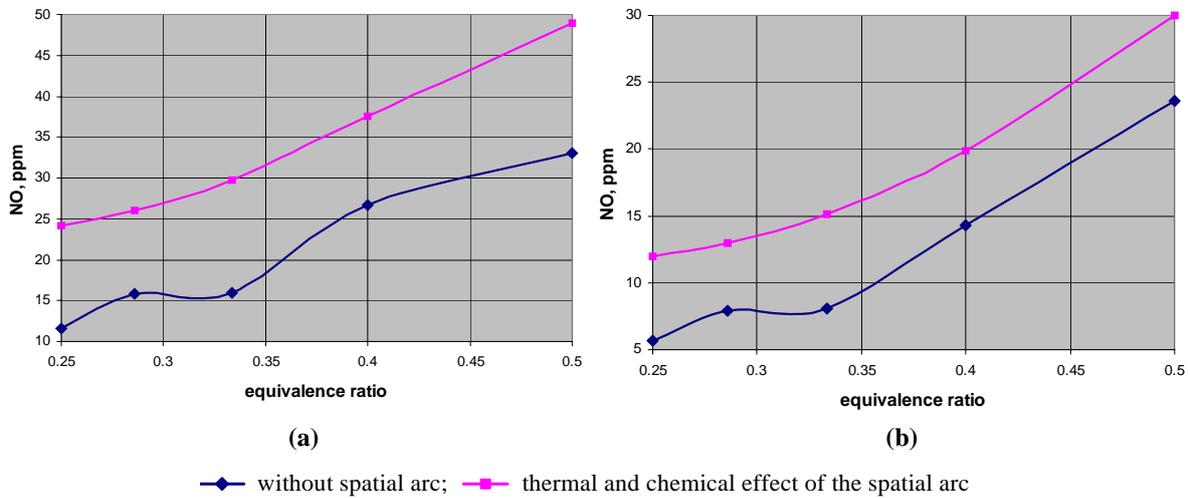


Figure 7. Spatial arc influence on exhaust NO emission in cross-section 1 (a) and 2 (b).

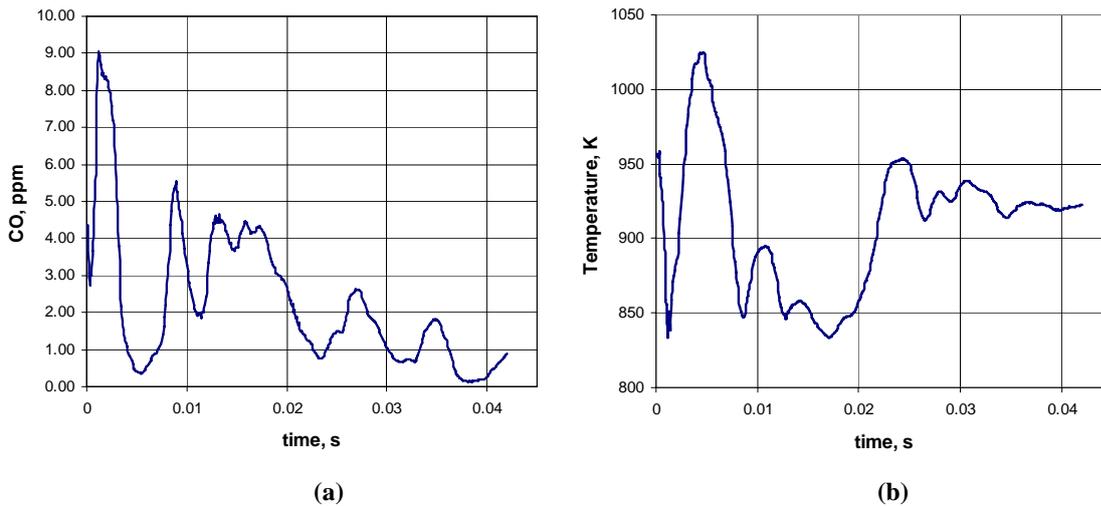


Figure 8. Unsteady LES-model calculations.

V. Conclusion

Theoretical investigations demonstrating the modeling opportunity for the complex aerodynamic flows in the “Tornado” combustor with spatial arc have been conducted. Relations between equivalence ratio, NO and CO emissions, and exhaust temperature were determined both for cases with and without spatial arc. Different factors of plasma influence such as thermal, chemical and mixing impact were analyzed. Numerical analysis allowed determine location of the pollutant forming and overheating zones inside the “Tornado” combustor.

Spatial arc application allows decrease the CO emission on 55 % for all fuel equivalence ratios in cross-section 1 and on 35 % in cross-section 2. The length of the active CO formation zone is reduced from 60 % for rich mixtures to 35 % for lean mixtures.

Obtained results and recommendations can be used for the reverse vortex combustor with spatial arc operational mode and geometry optimization, perspective combustors design and engineering for propulsion and power generation.

Successful completion of this work will result in a new combustion technology development for clean fuel burning in gas turbines for industrial (electricity and heat generation, pump drivers, etc.), marine (propulsion, electricity and heat production) applications, and ground transportation systems.

VI. References

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